MGSC 1205

Quantitative Methods I

Slides Two – Supply & Demand

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Demand Function

It is a function that relates the **price** of a product to the **quantity** of that product that consumers will purchase.

- *Demand* is a linear function, $D = f(p)$. Such a linear function can be written as:

$$D = \mathrm{m}p + \mathrm{B}$$

- where m is the ***slope*** or rate of change and B is the ***vertical intercept***.
- The slope is the change in quantity demanded per unit change in price (for each $1.00 increase in price).
- The intercept, B, tells us where the line crosses the *y*-axis. It gives the demand when price = $0.00.

Finding a Demand function given two observations

Example 1: Suppose that the demand is 4000 liters when the gas price is \$0.90 and it is 3800 liters when the gas price increased to \$1.00. Find the demand function of the gas station?

- *The demand* function is: $D = \mathrm{m}p + \mathrm{B}$
- The slope m = change in demand / change in price

$$m = (3800 – 4000)/(1.0 – 0.9) = - 2{,}000.$$

- The intercept B = D – m p

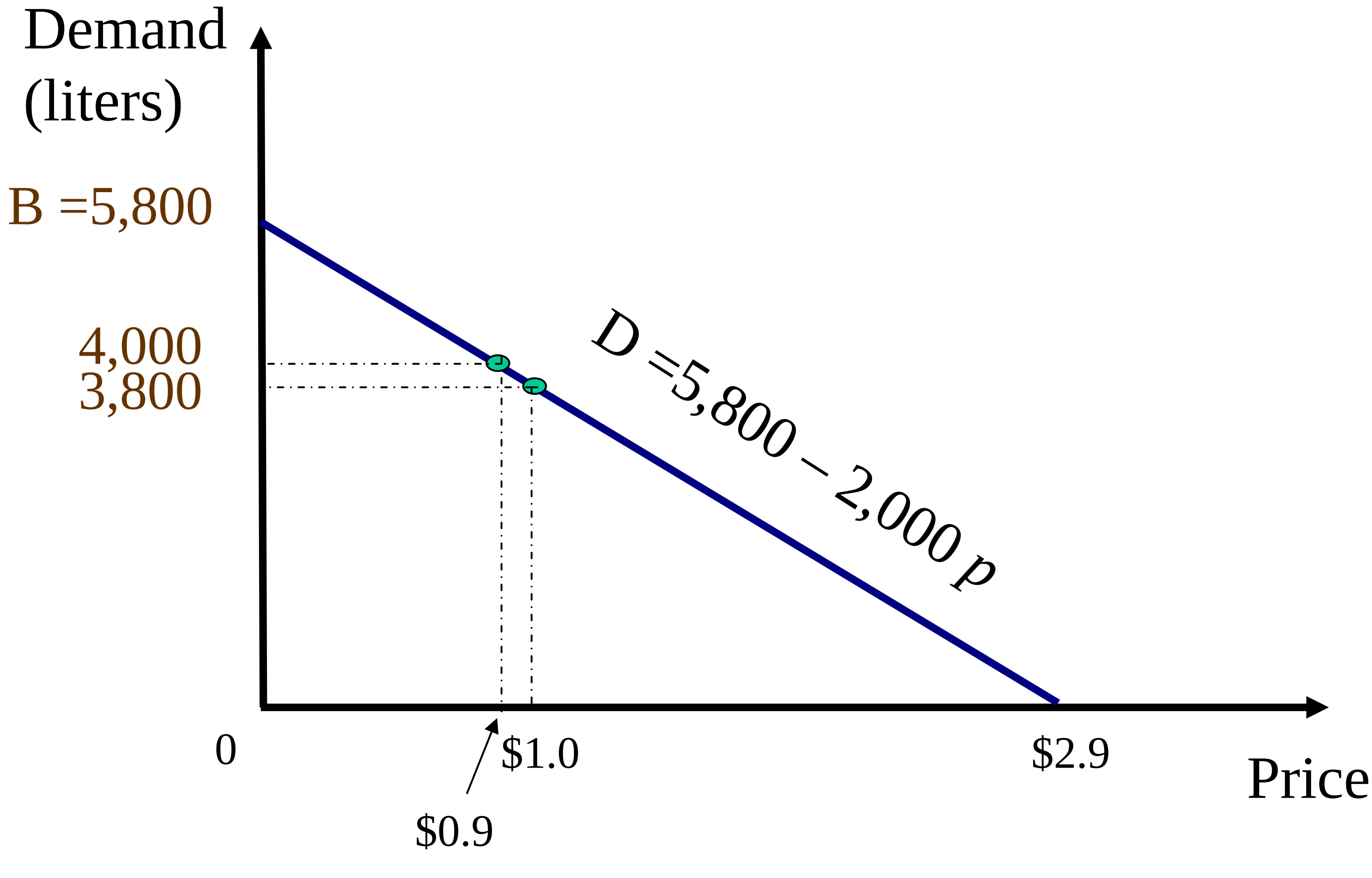
Using one the observed demand and price, say (3800, \$1),

$$B = 3800 – (- 2{,}000) * \$1.0 = 5{,}800$$

- Therefore, we have D =5,800 – 2,000 p.

$D = 5{,}800 - 2{,}000\, p$

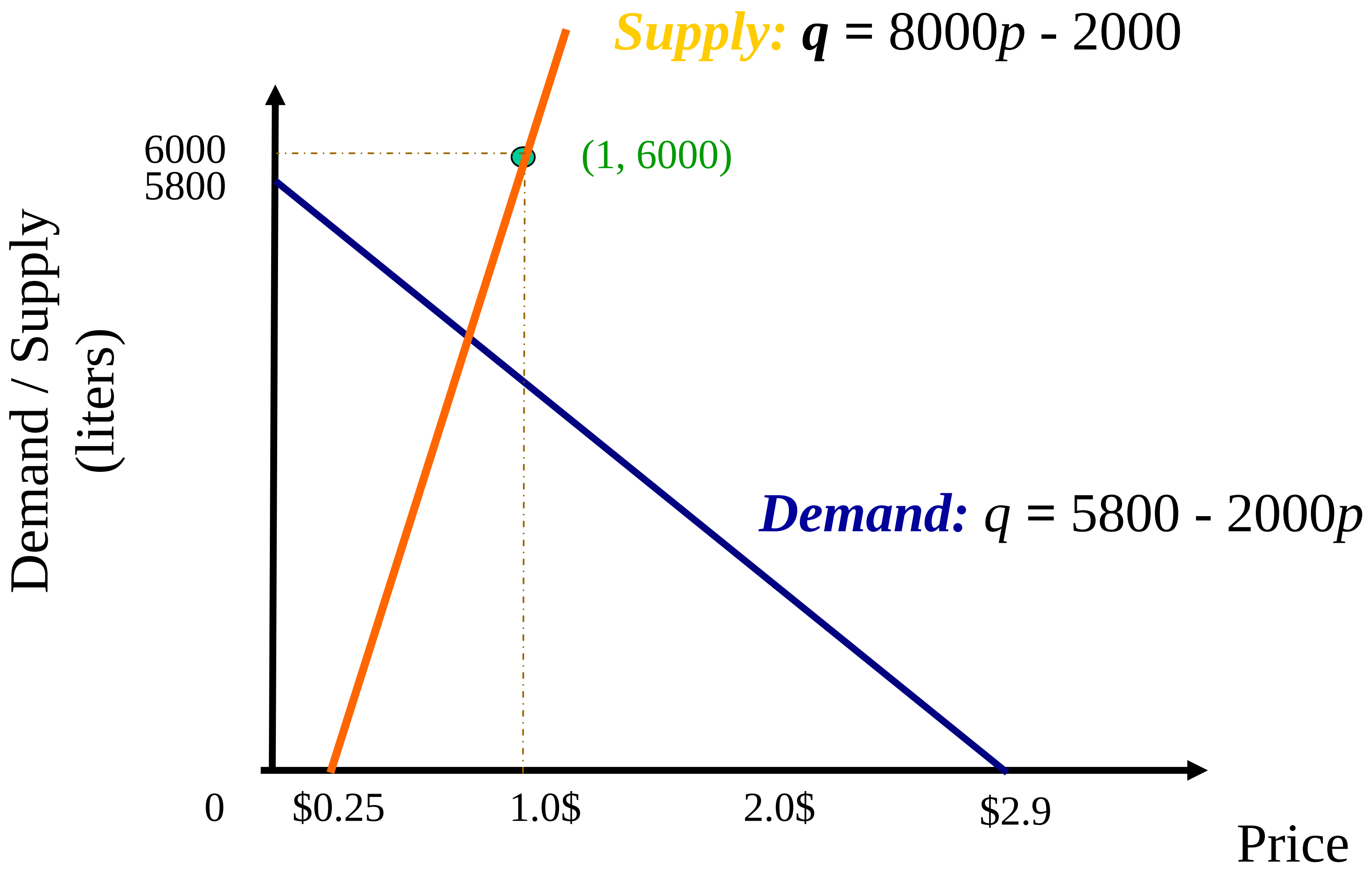
Demand	4000	3800	5,800	0
Gas price	$0.90	$1.00	$0.00	$2.90



Demand Function *vs.* Supply Function

- **Demand function:** a function that relates the **price** of a product to the **quantity** of that product that consumers will purchase.
  - Example: $q = 5800 – 2000\,p$
  - If prices are high, the demand will drop. If prices are decline, the demand will increase.
- **Supply function:** a function that relates the **price** of a product to the **quantity** of that product that manufacturers will produce.
  - If prices are high, the supply will increase. If prices are decline, the supply will drop.
  - Example: $q = 8000p - 2000$

Example

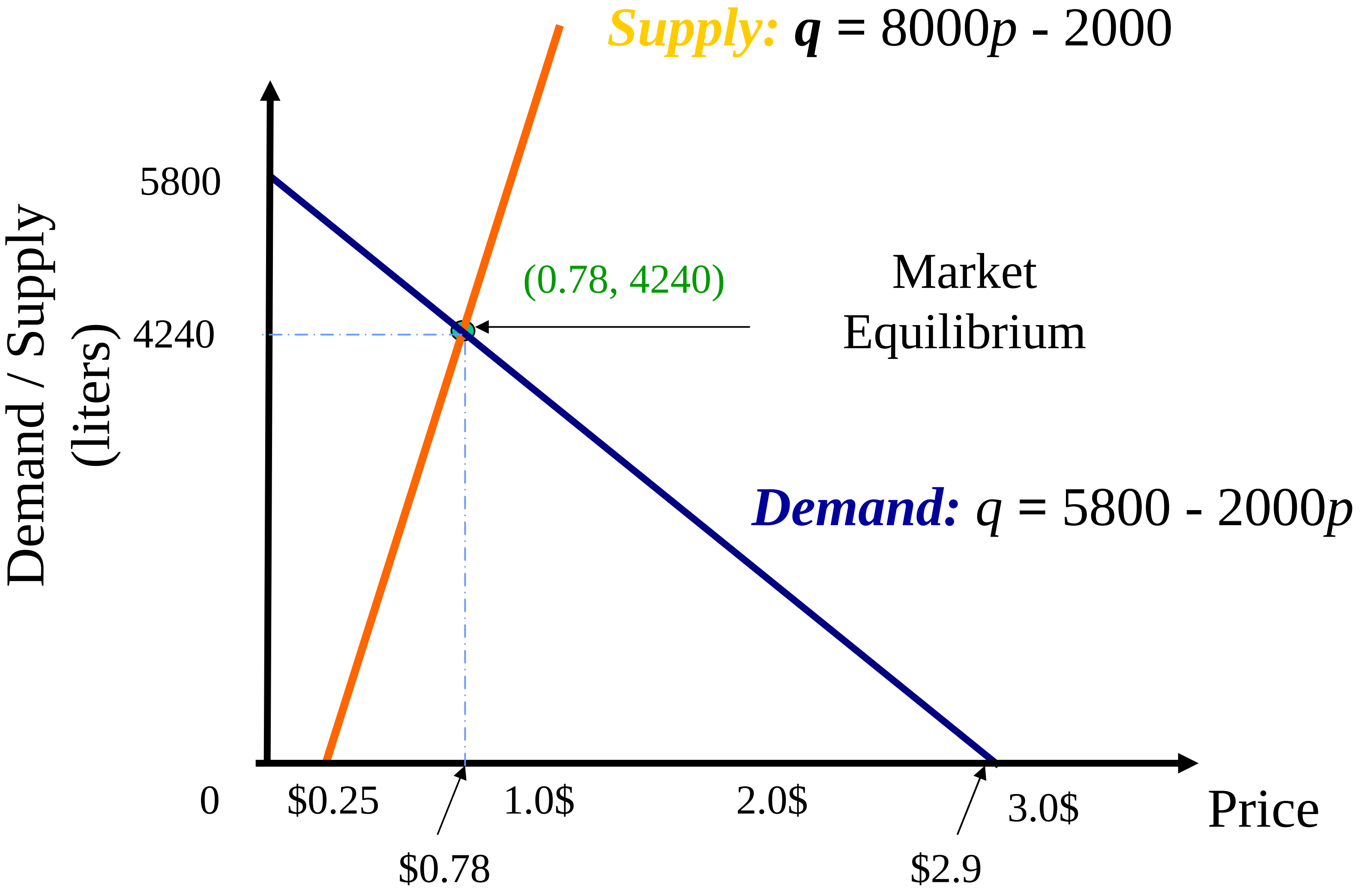


Supply: q = 8000p - 2000
6000
5800
(1, 6000)
Demand / Supply
(liters)
Demand: q = 5800 - 2000p
0
$0.25
1.0$
2.0$
$2.9
Price

Market Equilibrium

- Market equilibrium occurs when suppliers and consumers agree on a quantity that should be sold/bought.
- can be done several ways:
  - Graphically
  - Algebraically: set $D = S \rightarrow$ equilibrium price
  - Goal Seek in Excel
- Graphically, when the supply line crosses the demand line.

Example: Find the market equilibrium in the previous example.



Algebraic solving

- Set D = S
- 5800 – 2000p = 8000 p -2000
- p = 7800/10000 = $0.78
- D = S = 5800 – 2000 * (0.78) = 4240 liters.

Goal seek in Excel

- B3 : price ?
- B4 : Supply = 8000 * B3 -2000
- B5 : Demand = 5800 – 2000 * B3
- B6 : D – S = B5 – B4

➢ Use goal seek when
B6 = 0
by changing B3

Looking Deeper at Supply/Demand - Taxes

- What is the effect of raising taxes on a product?
- Suppose that the product is cigars and the provincial government has decided to levy a tax of 20%.
- From the supplier's perspective, their costs of production are still the same. So the supply function should not change.
- From the consumer's point of view, something that cost \$10 will now cost \$12, since the tax has effectively increased the price by 20%. So the demand function will change.

Example: Determine the market equilibrium price and quantity if
Demand function: quantity = 150 - 6*p
Supply function : quantity = -20 + 4*p

- Algebraic Solution:
  - Set D = S then
  - 150 – 6*p = -20 + 4*p then p = $17
  - D = 150 – 6*17 = 48 units.
  - Supplier revenue = units * price = 48*17 = $816
- Graphical method
- Goal seek method

Example: What is the new market equilibrium quantity and before-tax price if there is a tax of 20%.

- Tax rate = 20% = 0.20
- The new price: $p_{new} = p_{old} + 0.20^* p_{old} = 1.2^* p_{old}$
- The demand fn: $D = 150 – 6^*p_{new}$
  $D = 150 – 6^*1.2\ p_{old} = 150\text{-}7.2^*p_{old}$
- The supply fn: $S = -20 + 4\ p_{old}$
- **New market equilibrium (Algebrically):**
- ➢ Set D = S, then 150 – 7.2p = -20 + 4p
- ➢ 170 = (7.2 + 4) p
- ➢ Before-tax price: p = 170/11.2 = $15.18
- ➢ New market equilibrium quantity: D = 150-7.2*15.18 = 40.7
- ➢ Supplier revenue = D*p = 40.7*$15.18 = $617.83
- ➢ Tax revenue = tax* Demand

  = (supplier price * tax rate)*D

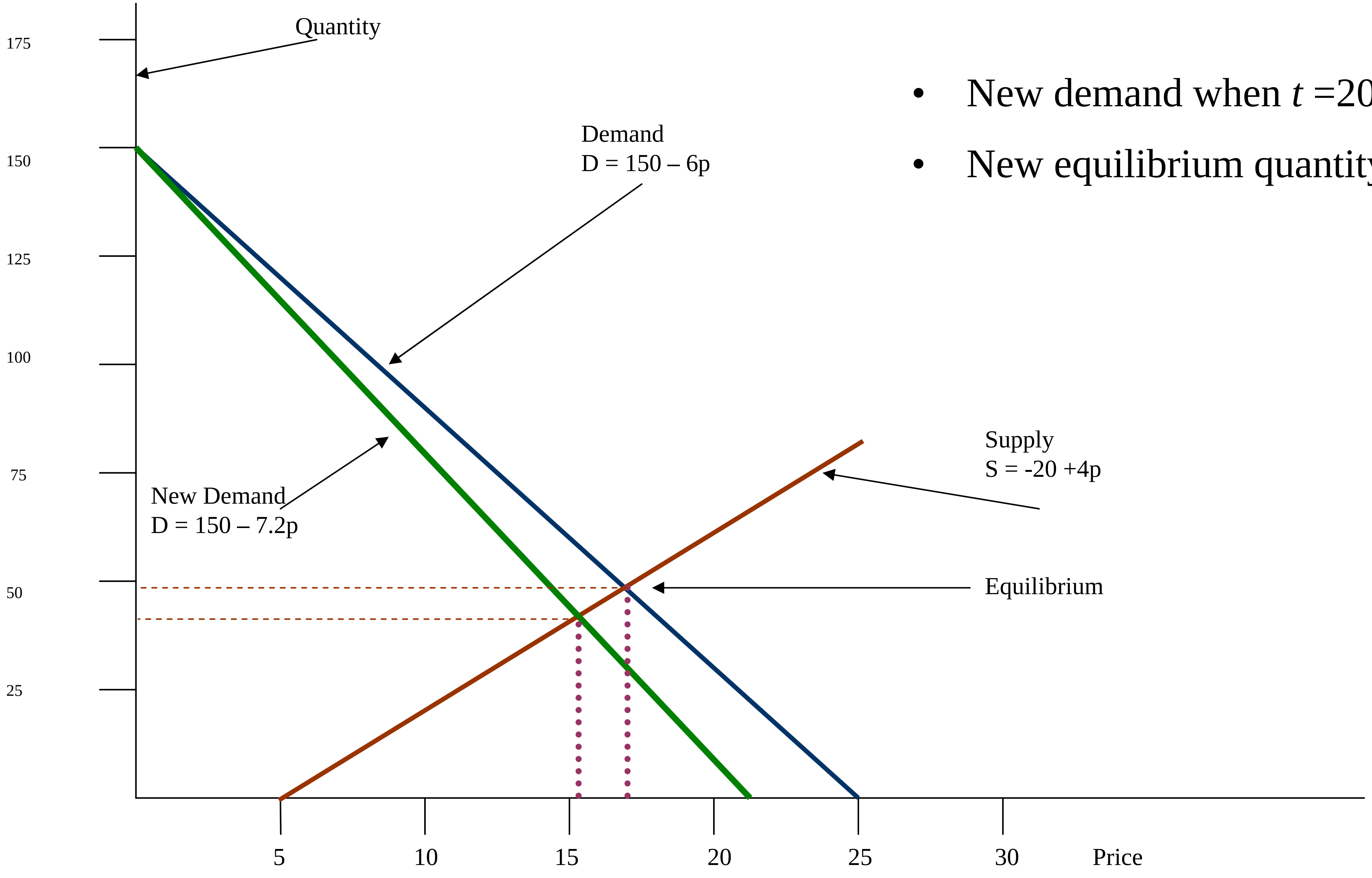
  = ($15.18*0.20)*40.7 = $3.04(40.7) = $123.56
- ➢ Total consumer expenditures = p_{new}*D

  = (1.2*$15.18)*40.7 = $741.39

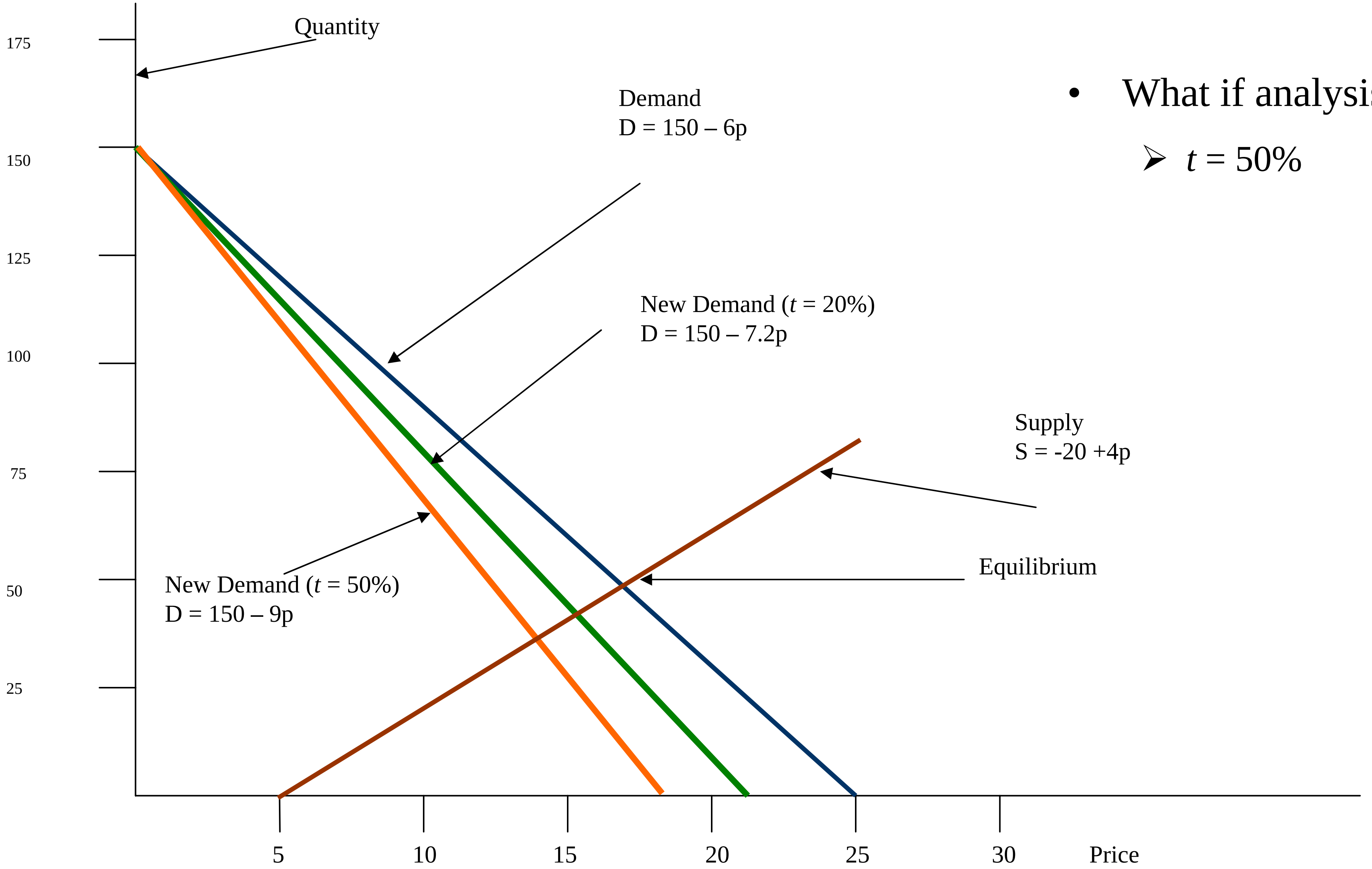
  = Supplier revenue + Tax revenue

Graphical method

- New demand when *t* =20%
- New equilibrium quantity



Effect of Taxes - What if Analysis



- What if analysis
  - t = 50%

Excel Model of Taxation

- Let us look at tax rates varying from 0.00 up to 1.00. This last value would correspond to a tax of 100%. The formulas for the various quantities we want to see are as follows:
- **A5** Tax rate **=** t **(**given**)**
- **B5** Supplier Price = 170/(10+6t)

  = 170/(10+6*A5)
- **C5** Consumer Price = supplier price plus taxes **= B5+(B5*A5)**
- **D5** Demand = 150 – 6*Consumer Price **= 150-6*C5**
- **E5** Supplier Revenue = Supplier Price * Demand **= B5*D5**
- **F5** Tax Revenue = Tax * Demand **= B5*A5*D5**
- **G5** Total Consumer Exp = Consumer Price * Demand

  = C5*D5

Microsoft Excel - Book2

B6 f_x =170/(10+6*A6)

	A	B	C	D	E	F	G
3							
4	Tax Rate	Supplier Price	Consumer Price	Demand	Supplier Revenue	Tax Revenue	Total Consumer Expenditures
5	0.00	$17.00	$17.00	48.00	$816.00	$0.00	$816.00
6	0.05	$16.50	$17.33	46.02	$759.54	$37.98	$797.52
7	0.10	$16.04	$17.64	44.15	$708.08	$70.81	$778.89
8	0.15	$15.60	$17.94	42.39	$661.06	$99.16	$760.21
9	0.20	$15.18	$18.21	40.71	$617.98	$123.60	$741.58
10	0.25	$14.78	$18.48	39.13	$578.45	$144.61	$723.06
11	0.30	$14.41	$18.73	37.63	$542.09	$162.63	$704.71
12	0.35	$14.05	$18.97	36.20	$508.57	$178.00	$686.57
13	0.40	$13.71	$19.19	34.84	$477.63	$191.05	$668.68
14	0.45	$13.39	$19.41	33.54	$449.00	$202.05	$651.06
15	0.50	$13.08	$19.62	32.31	$422.49	$211.24	$633.73
16	0.55	$12.78	$19.81	31.13	$397.87	$218.83	$616.71
17	0.60	$12.50	$20.00	30.00	$375.00	$225.00	$600.00
18	0.65	$12.23	$20.18	28.92	$353.71	$229.91	$583.62
19	0.70	$11.97	$20.35	27.89	$333.86	$233.70	$567.57
20	0.75	$11.72	$20.52	26.90	$315.34	$236.50	$551.84
21	0.80	$11.49	$20.68	25.95	$298.03	$238.42	$536.45
22	0.85	$11.26	$20.83	25.03	$281.83	$239.56	$521.39
23	0.90	$11.04	$20.97	24.16	$266.66	$239.99	$506.65
24	0.95	$10.83	$21.11	23.31	$252.42	$239.80	$492.23
25	1.00	$10.63	$21.25	22.50	$239.06	$239.06	$478.13

Sheet1 / Sheet2 / Sheet3

In this table you can see the many effects of taxation

- It drives up the price to consumers while reducing the price to suppliers.
- These effects drive down demand.
- The concurrent decreases in price and demand that suppliers see, very quickly drive down their revenues.
- Although consumers are paying more, their demand is decreasing and the total effect is a decrease in total expenditures.
- The spreadsheet is useful in showing all of these simultaneous effects.
- But remember, we couldn’t build this spreadsheet model without the algebraic solution to equilibrium.
- **The spreadsheet does not replace the need for mathematical skills, but it can add significant value to those fundamental skills.**